When viewed on a computer, this table of contents links to the various sections in the cheatsheet. If printed, it is usually not useful to include the table of contents.

Things to remember

- Use inertial reference frames
- Write physical laws (i.e. "Newton's 2nd Law" or "Work Kinetic Energy Theorem")

Kinematics Equations

Where v_0 is initial velocity, and a is acceleration. These equations can be used in any number of dimensions.

$$v_x(t) = v_{0x} + a_x t$$
$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$v_x^2(t) = v_{0x}^2 + 2a_x \times (x(t) - x_0)$$

Newton's Laws

1. **Law of Inertia:** A body on which zero net force acts either remains at rest or moves with constant velocity $\vec{v}$ in a straight line (i.e., $\vec{a} = 0$) This would be the natural "equilibrium" state of a body that is free from external force.

2. In an *inertial reference frame*, for each body

$$\sum \vec{F}_{\text{on body}} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \sum \vec{F}_{\text{on body}}$$

3. Force is the agent of *interaction* between two bodies. Each interaction between 2 bodies, A and B, consists of an *interaction pair* of "3rd law partner forces," $\vec{F}_{A \text{ on } B}$ and $\vec{F}_{B \text{ on } A}$

$$\vec{F}_{B \text{ on } A} = -\vec{F}_{A \text{ on } B}$$

This holds in any reference frame, intertial or non-inertial

PHYS 1112 Game Plan for Newton's Laws

1. Work in an inertial reference frame

2. Identify relevant interactions & forces (interaction type, direction, source body, subject body). Choose bodies or systems that enable you to "access" key forces acting across their boundaries.

3. For each body, draw a carefully labeled *free body diagram* showing only external interaction forces acting on that body. Use symbols for magnitude of force. Model the body as a particle.

4. Choose a convenient coordinate system for each body. Identify $+$ and $-$ directions.

5. Write out Newton's second law in components for each body with **interaction forces on the left side only**, using symbols from your free body diagram.

$$\sum_{\text{external}} \vec{F}_{\text{on body}} = m_{\text{body}}\vec{a}_{\text{body}}$$

6. Use the resulting equations to formulate answers

7. Check to make sure that your answers make physical sense.

Gravity

Gravity is a universal non-contact force. G is the universal constant of gravitation, and g is the acceleration due to gravity on earth.

$$G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$$
$$g = 9.80\,\text{N}\,\text{kg}^{-1} = 9.80\,\text{m}\,\text{s}^{-2}$$

Given two masses, m_1 and m_2, and the distance between them, r, the force of gravity is equal to

$$F_g = \frac{G \times m_1 \times m_2}{r^2} = m_1 \times g = w$$

Circular Motion

Circular motion involves two quantities, radial acceleration (a_{rad}) and tangential velocity (v_{tan}). Tangential velocity is the velocity tangent to the point on the circle that the mass is moving at, and radial acceleration is the acceleration towards the center of the circle.

$$a_{\text{rad}} = \frac{v_{\text{tan}}^2}{r}$$

Kinetic Energy

Kinetic energy is the amount of energy that is stored in the motion of an object. Kinetic energy can be computed as $\frac{1}{2}mv^2$, where m is the mass of the object, and v is the velocity.
Kinetic energy (and all other forms of energy) is measured in Joules.

Work

Work, W, is defined as $W = Fs$, F is the magnitude of the force, and s is the magnitude of the displacement vector. Work is measured in *joules*, where $1\,\text{J} = 1\,\text{N}\,\text{m} = 1\,\text{kg}\,\text{m}^2\,\text{s}^{-2}$.
The Joule is also the SI measurement of energy.

Work by Varying Forces

Work done by varying forces is defined as $W = \int_{x_1}^{x_2} F_x \, dx$, where x_1 is the initial position and x_2 is the final position.
Springs: The work done by a spring is $\frac{1}{2}kx^2$.
Motion along a curve: Where dW is the change in work, $\vec{F}$ is the force, $d\ell$ is the the curve between two points as a straight line.

$$dW = \vec{F} \cdot d\vec{\ell} = ||\vec{F}|| \cos \phi d\ell = F_{||} d\ell$$

From the textbook.

Power

Power is defined as the rate of change of work.

$$P = \lim_{\Delta \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

The SI unit of power is the *watt*. $1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}}$. Another common unit of power is the *horsepower*. $1\,\mathrm{hp} = 746\,\mathrm{W} = 0.746\,\mathrm{kW}$
The instantaneous power for a force doing work on a particle is equal to the dot product of force and velocity.

$$P = \vec{F} \cdot \vec{v}$$

Work-Kinetic Energy Theorem

The work-kinetic energy theorem states that the work done by the net force on a particle is equal to the change in the particle's kinetic energy:

$$W_{\text{total}} = K_2 - K_1 = \Delta K$$

Where W_{total} is the total work done on the particle, K_1 is the particle's initial kinetic energy, K_2 is the particle's final kinetic energy, and ΔK is the change in kinetic energy.

Gravitational Potential Energy

Gravitational potential energy of a particle is equal to mgh for objects near the surface of the earth, where m is the mass of the particle, g is the acceleration due to gravity, and t is the height of the particle. The change in gravitational potential energy is $mg(h_2 - h_1)$, where h_1 is the initial height, and h_2 is the final height. The zero height can be at whatever point you wish. Gravitational potential energy is denoted U_{grav}
For objects farther from the surface of the earth, gravitational potential energy is defined as, where G is the universal constant of gravitation, m_E is the mass of the earth, m is the mass of the object, and r is the distance from the earth's center to the object:

$$U = -\frac{G m_E m}{r}$$

For more information, see the section on gravitation.

Elastic Potential Energy

Elastic potential energy stored in an ideal spring is equal to $\frac{1}{2}kx^2$, where k is the spring constant, and x is spring's extension. If the spring is extended, $x > 0$, and if the spring is compressed, $x < 0$. Note that the $x = 0$ point is defined by the spring, and cannot be picked, unlike Gravitational Potential Energy. Elastic potential energy is denoted U_{el}.

Conservative and Non-Conservative Forces

A force that offers a conversion between gravitational and potential energy is called a *conservative force*. A force that does not offer the conversion is a *non-conservative force*.
Conservative forces are reversible, while non-conservative forces are not. A common conservative force is gravity, and a common non-conservative force is friction.

Conservation of Mechanical Energy

Conservation of mechanical energy states that $U_{\text{total}} + K_{\text{total}}$ is constant. Therefore we can say, for a system that only uses conservative forces:

$$K_1 + U_1 = K_2 + U_2$$

Momentum

Momentum is the vector quantity $\vec{p}$, where $\vec{p} = m\vec{v}$, where m is the mass of the particle and $\vec{v}$ is the velocity of the particle. Momentum has the same direction as velocity.
In terms of momentum, Newton's second law can be represented as:

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

In English, this means that the net external force on a particle is equal to the rate of change of the particle's momentum.

Impulse

The *impulse* of the net external force, denoted by $\vec{J}$, is the product of the net external force and the change in time.

$$\vec{J} = \sum \vec{F} \Delta t$$

Impulse is a vector quantity with the units Newton-seconds. $1\,\mathrm{N\,s} = 1\,\mathrm{kg\,m\,s^{-2}}$

Impulse-Momentum Theorem

The impulse of the net external force on a particle during a time interval is equal to the change in momentum of that particle during that interval.

$$\vec{J} = \vec{p_2} - \vec{p_1} = \Delta \vec{p}$$
$$\vec{J} = \vec{F}_{\text{avg}} \Delta t$$

Conservation of Momentum

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.
The total momentum of a system is equal to the vector sum of the momenta of all particles in the system.
You cannot find the magnitude of the total momentum in a system by adding the magnitudes of the momentum of each particle in a system.

Elasticity of Collisions

If the total mechanical energy in a system before and after a collision is equal, then the total kinetic energy in the system is also the same before and after the collision.
In an elastic collision, kinetic energy is conserved. In an inelastic collision, some kinetic energy is lost. In a completely inelastic collision, objects have the same final velocity (they travel together following the collision).

Elastic Collisions

If object b is at rest, and object a collides with it elastically, momentum and kinetic energy are conserved. Therefore, we can make the equations for kinetic energy and momentum respectfully.

$$\frac{1}{2}m_a v_{a1}^2 = \frac{1}{2}m_a v_{a2}^2 + \frac{1}{2}m_a = bv_{b2}^2$$

$$m_a v_{a1} = m_a v_{a2} + m_b v_{b2}$$

These equations can be simplified to:

$$v_{b2} = \frac{2m_a}{m_a + m_b} v_{a1}$$

in order to solve for the final velocity of object b.
When object b *isn't* initially at rest, you can adjust your reference frame to match the velocity of object b.
For all elastic collisions, $v_{b2} - v_{a2} = -(v_{b1} - v_{a1})$. In a straight-line elastic collision of two objects, the relative velocities before and after the collision have the same magnitude but opposite sign.

Completely Inelastic Collisions

In a completely inelastic collision, momentum is conserved but kinetic energy is not. However, because the bodies travel together following the collision, we can make the equation:

$$m_a v_{a1} + m_b v_{b1} = m_a v_2$$

Center of Mass (Particle Systems)

The *center of mass* of an object can be used to restate Conservation of Momentum in a useful way.
The center of mass of a system of n particles is defined as, where $\vec{r}$ is the position vector of the center of mass, m_i is the mass of particle i, and $\vec{r}_i$ is the position of particle ix

$$\vec{r} = \frac{\sum_i^n m_i \vec{r}_i}{\sum_i^n \vec{m}_i}$$

It is a *mass weighted average* of the positions of the particles.

Center of Mass (Objects)

For solid objects with continuous distribution of matter (constant density), if the object has a geometric center, the center of mass lies at the geometric center. If the object has an axis of symmetry, the center of mass lies on that object. The center of mass does not need to lie in the object, for example, a donut's center of mass is in the hole.

Motion of the Center of Mass

If there is no net external force on the center of mass, the velocity of the center of mass remains constant.
By repeatedly differentiating with respect to time, we can see that velocity, acceleration, and other similar quantities can be computed in a similar fashion to the position:

$$\vec{v} = \frac{\sum_i^n m_i \vec{v}_i}{\sum_i^n \vec{v}_i} \qquad \vec{a} = \frac{\sum_i^n m_i \vec{a}_i}{\sum_i^n \vec{a}_i}$$

The momentum of the center of mass is equal to the sum of the momentum of the particles. Where $\vec{p}$ is the total momentum of the system, M is the total mass of the system, and $\vec{v}_{\text{cm}}$ is the velocity of the center of mass:

$$M\vec{v}_{\text{cm}} = \sum_i^n m_i \vec{v}_i = \vec{P}$$

The ratio of the distance that each particle travels while the center of mass is maintained is equal to the *inverse* ratio of the masses of the objects.
If the net external force is not constant, we can state that, where M is the mass of the system, and $\vec{a}_{\text{cm}}$ is the acceleration of the center of mass:

$$\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}}$$

Rocket Propulsion

In rockets, we can't use newton's second law, $\sum \vec{F} = m\vec{a}$, because m changes. Rockets burn fuel to push themselves forward in space.
In rockets, where a is the acceleration of the rocket, $\frac{dv}{dt}$ is the rate of change in velocity (positive), $\frac{dm}{dt}$ is the rate of change of mass (negative), and v_{ex} is the exhaust speed (positive),

$$a = \frac{dv}{dt} = -\frac{v_{\text{ex}}}{m}\frac{dm}{dt}$$

From the textbook.

Angular Velocity

A quick radian review! A full circle is 2π radians. Where s is the arc length, and r is the radius, the central angle measure in radians is $\theta = \frac{s}{r}$ or $s = r\theta$. Trig cheatsheet from Lamar University.

The *average angular velocity*, $\omega_{\text{av}-z}$, is defined as

$$\omega_{\text{av}-z} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

The *instantaneous angular velocity* is defined as

$$\omega_z = \lim_{\Delta \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Conventionally, the positive direction for ω_z is the counterclockwise direction.

Angular velocity can also be represented as a vector. You can use the right-hand rule (figure 2) to determine the direction of the vector. Use your right hand. No, seriously, Will, I know you're going to use your left hand on the test, but don't. Check to make sure, remember, your left hand makes an L. Curl your fingers in the direction of rotation. Your thumb will point in the direction of $\vec{\omega}$.

Angular Acceleration

Angular acceleration can be computed in a very similar matter to angular velocity. It is generally denoted by α.

$$\alpha_{\text{av}-z} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \qquad \alpha_z = \lim_{\Delta \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

When the speed of rotation is increasing, the sign of $\vec{\omega}$ and $\vec{\alpha}$ are the same. When the speed of rotation is decreasing, $\vec{\omega}$ and $\vec{\alpha}$ have opposite signs.

Rotational Kinematics

Kinematics of rotational motion are very similar to the kinematics of linear motion

In fixed-axis rotation with constant angular acceleration, α_z:

$$\omega_z = \omega_{0z} + \alpha_z t$$

$$\theta = \theta_0 + \omega_{0z} t + \frac{1}{2}\alpha t^2$$

$$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$$

$$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$$

Relating Linear and Rotational Kinematics

The linear speed, v of a point on a rotating body, where r is the distance from the axis of rotation, and ω is the angular speed is:

$$v = r\omega$$

Tangential acceleration, a, of a point on a rotating body, where α is the radial acceleration is

$$a_{\text{tan}} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$$

The centripetal acceleration, towards the axis of rotation, of a given point is:

$$a_{\text{rad}} = \frac{v^2}{r} = \omega^2 r$$

The above equations only work in radians.

Energy in Rotational Motion

The *moment of inertia*, I, of a body for a given rotation axis is defined as:

$$I = \sum_i m_i r_i^2$$

Moment here means that I depends on how the body's mass is distributed in space. The greater the distance from the axis to the particles, the greater the moment of inertia. The SI unit of moments of inertia are kilogram-meters squared.

The rotational kinetic energy, K, of a rigid body rotating around an axis, where I is the moment of inertia, and ω is the angular velocity, is equal to

$$K = \frac{1}{2}I\omega^2$$

Calculating Moments of Inertia

To calculate a moment of inertia, take the integral:

$$I = \int_{R_1}^{R_2} \rho r^2 \, dm$$

See the textbook, or figure 1 for a list of common moments of inertia.

Parallel Axis Theorem

For a moment of inertia I_P a distance d from and parallel to a moment of inertia through the center of mass, I_{CM}, the moment of inertia is equal to

$$I_P = I_{\text{CM}} + Md^2$$

Torque

Torque, represented as τ, is the twisting effort of a specific force. It is calculated, where τ is torque, F is the perpendicular force to the lever arm, and l is the length of the lever arm as:

$$\tau = lF$$

The SI unit for torque is the *newton-meter*.

As a vector, torque can be represented as $\vec{\tau} = \vec{r} \times \vec{F}$, where $\vec{r}$ is the vector from the point about which rotation is occurring to the point where the force is being applied. You can use the right-hand rule to determine the direction of the torque vector.

Torque and Angular Rotation

Net torque on a particle causes angular acceleration about the particle.

$$\sum \tau_z = I_z \alpha_z$$

When torques are equal and in opposite directions, the net torque is zero along any axis.
When translation and rotation are combined in a body, the total kinetic energy of the body can be calculated as:

$$K = \frac{1}{2} M v_{\text{cm}}^2 + \frac{1}{2} I_{\text{cm}} \omega^2$$

A very common case of this is *rolling without slipping*. For an object to roll without slipping, v_{cm}, the speed of the center of mass of the rolling wheel must equal R, the radius of the wheel, times ω, the angular speed of the wheel: $v_{\text{cm}} = R\omega$

Torque and Work/Energy

The work done by a torque, τ_z is equal to the integral over the angular rotation of the torque:

$$W = \int_{\theta_1}^{\theta_2} \tau_z d\theta$$

If torque remains constant, the work equation simplifies to $W = \tau_z \Delta\theta$. If τ is represented in newton-meters and θ is represented in radians, the work is in joules.
Power due to torque on a rigid body is equal to:

$$P = \tau_z \omega_z$$

Remember, work is equal to the change in kinetic energy, so:

$$W_{\text{tot}} = \int_{\omega_1}^{\omega_2} I \omega_z d\omega_z = \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2$$

Angular Momentum

The angular momentum, $\vec{L}$ about an axis of symmetry for a given body is

$$\vec{L} = I\vec{\omega}$$

For a system of particles, the net torque on the system is equal to the rate of change of the total angular momentum of the system:

$$\sum \tau = \frac{d\vec{L}}{dt}$$

Conservation of Angular Momentum

When the net external torque acting on a system is zero, the total angular momentum of the system is constant conserved.

$$I_1 \omega_{1z} = I_2 \omega_{2z} \text{ (for no net external torque)}$$

Gravitation

Newton's law of universal gravitation states that "every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them." That is, for the universal gravitaion constant G, and particles with masses m_1 and m_2, r distance apart, the force due to gravity, F_g, is:

$$F_g = \frac{G m_1 m_2}{r^2}$$

The accepted value for G is $6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$.
We can use Newton's law of universal gravitation to define *weight* in terms of G, the radius of the earth R_E, the mass of the earth m_E, and the mass of the object, m:

$$w = F_g = \frac{G m_E m}{R_E^2}$$

Furthermore, the acceleration due to gravity is:

$$g = \frac{G m_E}{R_E^2}$$

Satellite Motion

The speed of a satellite in circular orbit around the earth is equal to

$$v = \sqrt{\frac{G m_E}{r}}$$

The period of a circular orbit is

$$T = \frac{2\pi r}{v} = 2\pi r \sqrt{\frac{r}{G m_e}} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{G m_E}}$$

The total mechanical energy of a circular orbit is

$$E = K + U = -\frac{G m_E m}{2r}$$

Other possibly useful information

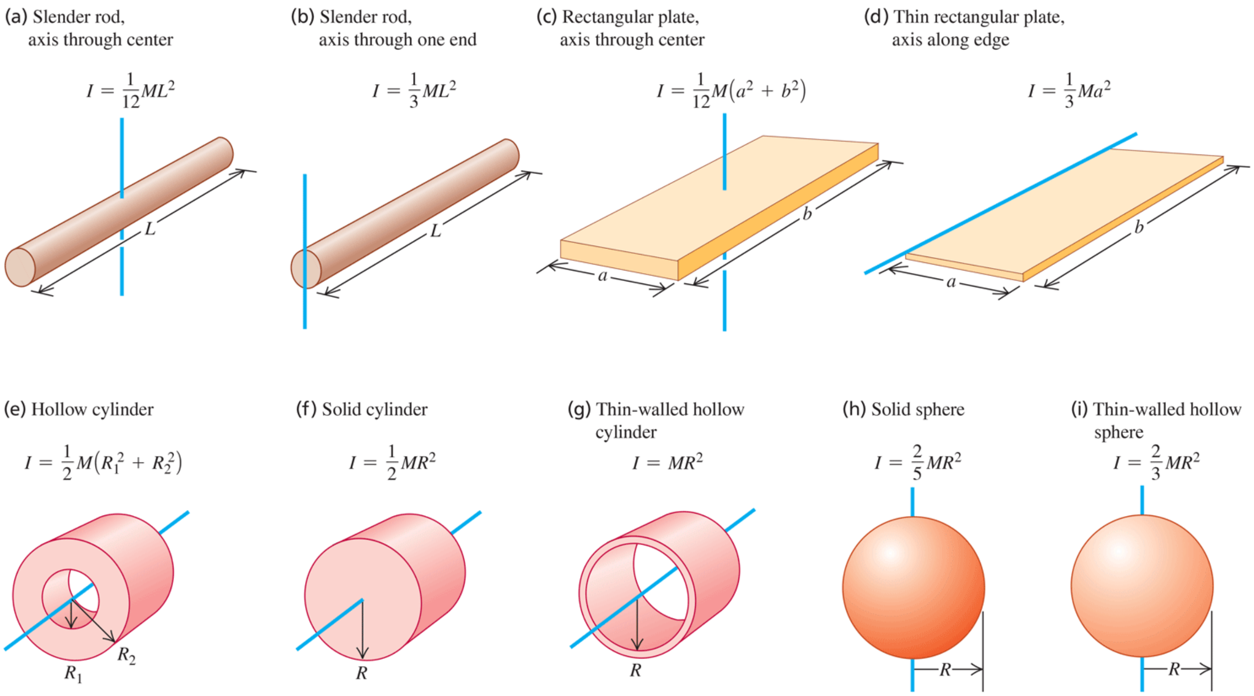

Figure 1: Moments of inertia for various common shapes

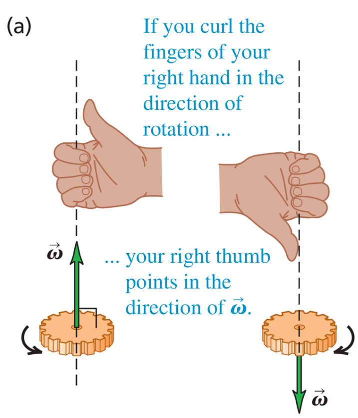

Figure 2: Right hand rule for rotational vectors

Figure 3: Cute puppies to look at when you're stressed